Answers and Explanations

1	c	**2**	c	**3**	a	**4**	d	**5**	d	**6**	a	**7**	c	**8**	a	**9**	d	**10**	b
11	a	**12**	d	**13**	a	**14**	b	**15**	a	**16**	d	**17**	d	**18**	a	**19**	c	**20**	b
21	c	**22**	b	**23**	a	**24**	a	**25**	d	**26**	b	**27**	d	**28**	c	**29**	b	**30**	d
31	c	**32**	d	**33**	c	**34**	c	**35**	d	**36**	a	**37**	b	**38**	c	**39**	c	**40**	b
41	a	**42**	b	**43**	d	**44**	b	**45**	a	**46**	c	**47**	a	**48**	c	**49**	b	**50**	a
51	b	**52**	c	**53**	c	**54**	c	**55**	a	**56**	c	**57**	c	**58**	a	**59**	b	**60**	d
61	b	**62**	b	**63**	b	**64**	c	**65**	d	**66**	a	**67**	b	**68**	d	**69**	d	**70**	b
71	c	**72**	b	**73**	a	**74**	a	**75**	b	**76**	d	**77**	d						

1. c Let the radius of the outer circle be x = OQ

Hence, perimeter of the circle $= 2\pi x$

But OQ = BC = x (diagonals of the square BQCO)

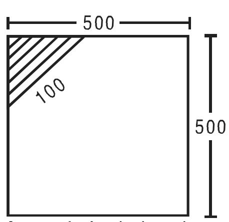
Perimeter of ABCD = 4x

Hence, ratio = $\frac{2\pi x}{4x} = \frac{\pi}{2}$.

2. c Following rule should be used in this case: The perimeter of any polygon circumscribed about a circle is always greater than the circumference of the circle and the perimeter of any polygon inscribed in a circle is always less than the circumference of the circle. Since, the circles is of radius 1, its circumference will be 2π. Hence, L1(13) > 2 π and L2(17) < π.

So {L1(13) + 2π } > 4 and hence $\frac{\{L1(13) + 2\pi\}}{L2(17)}$ will be greater than 2.

3. a



Area of shaded region

$= \frac{1}{2} \times \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}}$ = 2,500 sq m

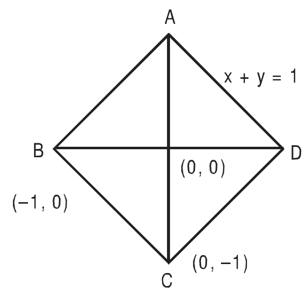
Area of a Δ is maximum when it is an isosceles Δ.

So perpendicular sides should be of length $\frac{100}{\sqrt{2}}$.

4. d We have not been given the distances between any two points.

5. d Since CD > DE, option (b) cannot be the answer. Similarly, since AB > AF, Option (c) cannot be the answer. We are not sure about the positions of points B and F. Hence, (a) cannot be the answer.

6. a The gradient of the line AD is –1. Coordinates of B are (– 1, 0).



Equation of line BC is x + y = –1.

7. c Let the area of sector S_1 be x units. Then the area of the corresponding sectors shall be 2x, 4x, 8x,16x, 32x and 64x. Since every successive sector has an angle that is twice the previous one, the total area then shall be 127x units. This is $\frac{1}{8}$ of the total area of the circle.

Hence, the total area of the circle will be 127x × 8

= 1016x units. Hence, angle of sector S_1 is $\frac{\pi}{1016}$.

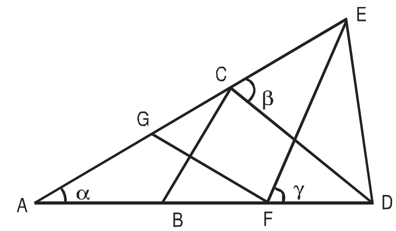
8. a We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 3ab + 3bc + 3ac$

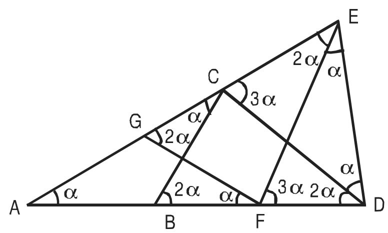
Now assume values of a, b, c and substitute in this equation to check the options.

Short cut: $(a – b)^2 + (b – c)^2 + (c – a)^2 = 0$.

Hence, a = b = c.

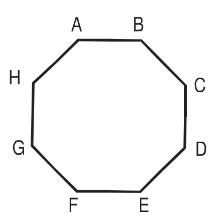
9. d





Let ∠EAD = α. Then ∠AFG = α and also ∠ACB = α.
Therefore, ∠CBD = 2α (exterior angle to ΔABC).
Also ∠CDB = 2α (since CB = CD).
Further, ∠FGC = 2α (exterior angle to ΔAFG).
Since GF = EF, ∠FEG = 2α. Now ∠DCE = ∠DEC = β (say). Then ∠DEF = β – 2α.
Note that ∠DCB = 180 – (α + β).
Therefore, in ΔDCB, 180 – (α + β) + 2α + 2α = 180 or β = 3α. Further ∠EFD = ∠EDF = γ (say).
Then ∠EDC = γ – 2α. If CD and EF meet at P, then ∠FPD = 180 – 5α (because β = 3α).
Now in ΔPFD, 180 – 5α + γ + 2α = 180 or γ = 3α.
Therefore, in ΔEFD, α + 2γ = 180 or α + 6α = 180 or α = 26 or approximately 25.

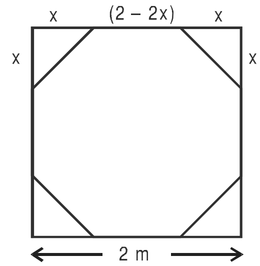
10. a



In order to reach E from A, it can walk clockwise as well as anticlockwise. In all cases, it will have to take odd number of jumps from one vertex to another. But the sum will be even. In simple case, if n = 4, then a_n = 2. For a_{2n-1} = 7 (odd), we cannot reach the point E.

11. d Work with options. Length of wire must be a multiple of 6 and 8. Number of poles should be one more than the multiple.

12. a

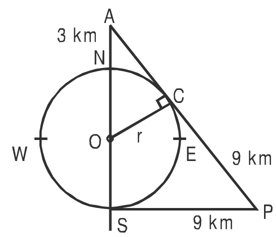


Let the length of the edge cut at each corner be x m. Since the resulting figure is a regular octagon,

$$\therefore \sqrt{x^2 + x^2} = 2 - 2x \Rightarrow x\sqrt{2} = 2 - 2x$$

$$\Rightarrow \sqrt{2}\,x\ (1 + \sqrt{2}) = 2 \Rightarrow x = \frac{\sqrt{2}}{\sqrt{2}+1}$$

13. b



ΔAPS and ΔAOC are similar triangles.
Where OC = r

$$\therefore \frac{r}{r+3} = \frac{9}{\sqrt{81 + (2r + 3)^2}}$$

Now use the options. Hence, the diameter is 9 km.

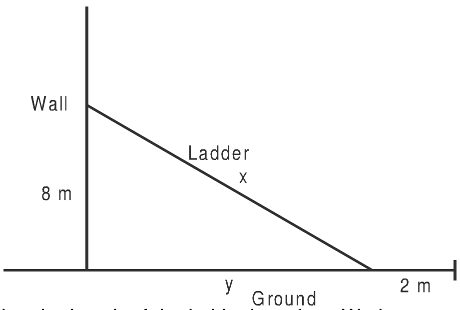
14. a Let BC = y and AB = x.
Then area of ΔCEF = Area(ΔCEB) – Area(ΔCFB)

$$= \frac{1}{2}.\frac{2x}{3}.y - \frac{1}{2}.\frac{x}{3}.y = \frac{xy}{6}$$

Area of ABCD = xy
∴ Ratio of area of ΔCEF and area of ABCD is

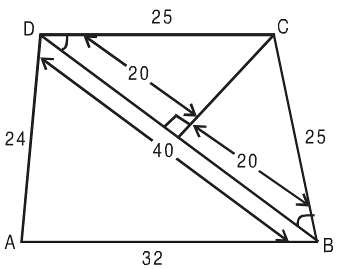
$$\frac{xy}{6} : xy = \frac{1}{6}$$

15. d



Let the length of the ladder be x feet. We have
$8^2 + y^2 = x^2$ and $(y + 2) = x$
Hence, $64 + (x - 2)^2 = x^2$
$\Rightarrow 64 + x^2 - 4x + 4 = x^2$
$\Rightarrow 68 = 4x \Rightarrow x = 17$

16. d



$CE = \sqrt{25^2 - 20^2} = 15$

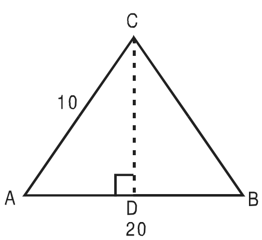
(Since DBC is isosceles triangle.)
Assume ABCD is a quadrilateral where AB = 32 m, AD = 24 m, DC = 25 m, CB = 25 m and $\angle DAB$ is right angle.
Then DB = 40 m because ΔADB is a right-angled triangle and DBC is an isosceles triangle.

So area of $\Delta\ ADB = \frac{1}{2} \times 32 \times 24 = 384$ sq. m

Area of $\Delta\ BCD = 2 \times \frac{1}{2} \times 15 \times 20 = 300$ sq. m

Hence area of ABCD = 384 + 300 = 684 sq. m

17. a



Let's assume AB be the longest side of 20 unit and another side AC is 10 unit. Here CD $\perp$ AB.

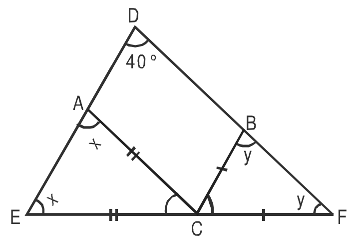
Since area of $\Delta ABC = 80 = \frac{1}{2} AB \times CD$

So $CD = \frac{80 \times 2}{20} = 8$. In ΔACD; $AD = \sqrt{10^2 - 8^2} = 6$

Hence DB = 20 – 6 = 14.

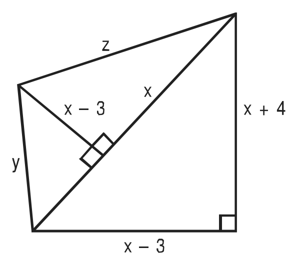
So $CB = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260}$ unit

18. c



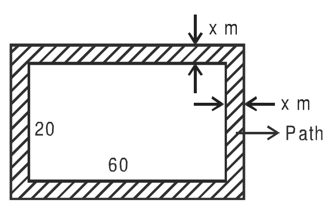
Here $\angle ACE = 180 - 2x$, $\angle BCF = 180 - 2y$
and $x + y + 40° = 180°$ (In ΔDEF)
So $x + y = 140°$
So $\angle ACB = 180° - \angle ACE - \angle BCF$
$= 180° - (180° - 2x) - (180° - 2y)$
$= 2(x + y) - 180°$
$= 2 \times 140 - 180 = 100°$

19. b



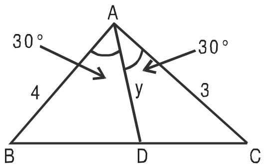
We can find the value of x, using the answer choices given in the question. We put (a), (b), (c) and (d) individually in the figure and find out the consistency of the figure. Only (b), i.e. 11 is consistent with the figure.

20. c



Let width of the path be x metres.
Then area of the path = 516 sq. m
$\Rightarrow (60 + 2x)(20 + 2x) - 60 \times 20 = 516$
$\Rightarrow 1200 + 120x + 40x + 4x^2 - 1200 = 516$
$\Rightarrow 4x^2 + 160x - 516 = 0 \Rightarrow x^2 + 40x - 129 = 0$
Using the answer choices, we get x = 3.

21. b



Let BC = x and AD = y.

As per bisector theorem, $\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3}$

Hence, BD = $\frac{4x}{7}$; DC = $\frac{3x}{7}$

In ΔABD, $\cos 30° = \dfrac{(4)^2 + y^2 - \frac{16x^2}{49}}{2 \times 4 \times y}$

$\Rightarrow 2 \times 4 \times y \times \frac{\sqrt{3}}{2} = 16 + y^2 - \frac{16x^2}{49}$

$\Rightarrow 4\sqrt{3}y = 16 + y^2 - \frac{16x^2}{49}$ ... (i)

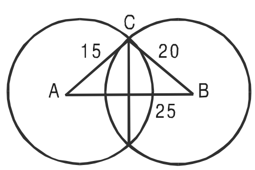
Similarly, from ΔADC, $\cos 30° = \dfrac{9 + y^2 - \frac{9x^2}{49}}{2 \times 3 \times y}$

$\Rightarrow 3\sqrt{3}y = 9 + y^2 - \frac{9x^2}{49}$ ... (ii)

Now (i) × 9 – 16 × (ii), we get

$36\sqrt{3}y - 48\sqrt{3}y = 9y^2 - 16y^2 \Rightarrow y = \frac{12\sqrt{3}}{7}$

22. a



Let the chord = x cm

$\therefore \frac{1}{2}(15 \times 20) = \frac{1}{2} \times 25 \times \frac{x}{2} \Rightarrow x = 24$ cm

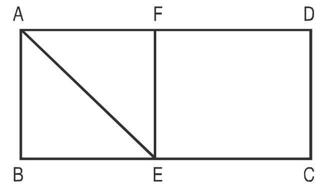
23. a Total area = 14 × 14 = 196 m^2

Grazed area = $\left(\frac{\pi \times r^2}{4}\right) \times 4 = \pi r^2$ = 22 × 7(r = 7)

= 154 m^2

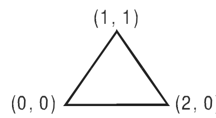
Ungrazed area is less than (196 – 154) = 42 m^2, for which there is only one option.

24. d



Area of ΔABE = 7 cm^2
Area of ΔABEF = 14 cm^2
Area of ΔABCD = 14 × 4 = 56 cm^2

25. b



Let a = 0

Hence, area = $\frac{1}{2}(2)\ (1) = 1$

Note: Answer should be independent of a and area of the triangle does not have square root.

26. d Check choices, e.g. $\frac{1}{2} \Rightarrow$ Diagonal = $\sqrt{5}$

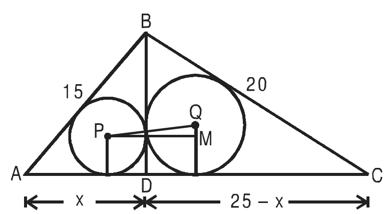
Distance saved = $3 - \sqrt{5} \approx 0.75 \neq$ Half the larger side.
Hence, incorrect.

$\frac{3}{4} \Rightarrow$ Diagonal = 5

Distance saved = (4 + 3) – 5 = 2 = Half the larger side.

27. c Area = 40 × 20 = 800
If 3 rounds are done, area = 34 × 14 = 476
$\Rightarrow$ Area > 3 rounds
If 4 rounds $\Rightarrow$ Area left = 32 × 12 = 347
Hence, area should be slightly less than 4 rounds.

28. b



$(15)^2 - x^2 = (20)^2 - (25 - x)^2$

$\Rightarrow x = 9$

$\Rightarrow BD = 12$

Area of $\Delta ABD = \frac{1}{2} \times 12 \times 9 = 54$

$s = \frac{1}{2}(15 + 12 + 9) = 18$

$r_1 = \frac{\text{Area}}{s} \Rightarrow r_1 = 3$

Area of $\Delta BCD = \frac{1}{2}\times 16 \times 12 = 96$

$s = \frac{1}{2}(16+20+12) = 24$

$r_2 = \frac{Area}{s} \Rightarrow r_2 = 4$

In ΔPQM, $PM = r_1 + r_2 = 7$ cm
$QM = r_2 - r_1 = 1$ cm

Hence, $PQ = \sqrt{50}$ cm

29. d If KL = 1, then IG = 1 and FI = 2

Hence, $\tan\theta = \frac{2}{1} = 2$

Thus, θ none of 30, 45 and 60°.

30. c Area of quadrilateral ABCD $= \frac{1}{2}(2x+4x)\times 4x = 12x$

Area of quadrilateral DEFG $= \frac{1}{2}(5x+2x)\times 2x = 7x$

Hence, ratio = 12 : 7

31. d The surface area of a sphere is proportional to the square of the radius.

Thus, $\frac{S_B}{S_A} = \frac{4}{1}$ (S. A. of B is 300% higher than A)

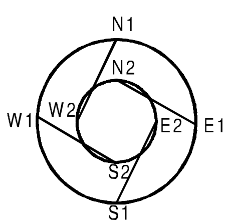
$\therefore \frac{r_B}{r_A} = \frac{2}{1}$

The volume of a sphere is proportional to the cube of the radius.

Thus, $\frac{V_B}{V_A} = \frac{8}{1}$

Or, V_A is $\frac{7}{8}$th less than B i.e. $\left(\frac{7}{8}\times 100\right)$ 87.5%

For questions 32 to 34:



If the radius of the inner ring road is r, then the radius of the outer ring road will be 2r (since the circumference is double).

The length of IR = 2π r, that of OR = 4π r and that of the chord roads are $r\sqrt{5}$ (Pythagoras theorem)

The corresponding speeds are

20π, 30π and $15\sqrt{5}$ kmph.

Thus time taken to travel one circumference of

IR = $\frac{r}{10}$ hr, one circumference of OR = $\frac{r}{7.5}$ hr hr.

and one length of the chord road = $\frac{r}{15}$ hr

32. c Sum of the length of the chord roads = $4r\sqrt{5}$ and the length of OR = 4π r.

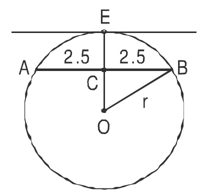
Thus the required ratio = $\sqrt{5} : \pi$

33. c The total time taken by the route given = $\frac{r}{30}+\frac{r}{15}=\frac{3}{2}$
(i.e. 90 min.)
Thus, r = 15 km. The radius of OR = 2r = 30 kms

34. d The total time taken = $\frac{r}{20}+\frac{r}{15}=\frac{7r}{60}$

Since r = 15, total time taken = $\frac{7}{4}$ hr. = 105 min.

35. a



We can get the answer using the second statement only. Let the radius be r.
AC = CB = 2.5 and using statement B, CE = 5, thus OC = (r – 5).
Using Pythagoras theorem, $(r-5)^2 + (2.5)^2 = r^2$
We get r = 3.125

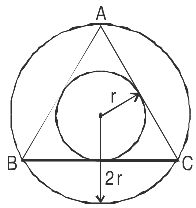
NOTE: You will realize that such a circle is not possible (if r = 3.125 how can CE be 5). However we need to check data sufficiency and not data consistency. Since we are able to find the value of r uniquely using second statement the answer is (a).

36. b The question tells us that the area of triangle DEF will be $\frac{1}{4}$ th the area of triangle ABC. Thus by knowing either of the statements, we get the area of the triangle DEF.

37. c In this kind of polygon, the number of convex angles will always be exactly 4 more than the number of concave angles.

NOTE : The number of vertices have to be even. Hence the number of concave and convex corners should add up to an even number. This is true only for the answer choice (c).

38. c



Since the area of the outer circle is 4 times the area of the inner circle, the radius of the outer circle should be 2 times that of the inner circle.
Since AB and AC are the tangents to the inner circle, they should be equal. Also, BC should be a tangent to inner circle. In other words, triangle ABC should be equilateral.
The area of the outer circle is 12. Hence the area of inner circle is 3 or the radius is $\sqrt{\frac{3}{\pi}}$. The area of equilateral triangle $= 3\sqrt{3}\ r^2$, where r is the inradius.

Hence the answer is $\frac{9\sqrt{3}}{\pi}$

39. b If the radius of the field is r, then the total area of the field = $\frac{\pi r^2}{2}$.
The radius of the semi-circles with centre's P and $R = \frac{r}{2}$.

Hence, their total area = $\frac{\pi r^2}{4}$

Let the radius if the circle with centre S be x.

Thus, OS = (r – x), OR = $\frac{r}{2}$ and $RS = \left(\frac{r}{2} + x\right)$.

Applying Pythagoras theorem, we get

$(r - x)^2 + \ -^2 \quad -^2$

$x = \frac{r}{3}$.

Thus the area of the circle with centre $S = \frac{\pi r^2}{9}$.

The total area that can be grazed = $\pi r^2\left(\frac{1}{4} + \frac{1}{9}\right)$

$= \frac{13\pi r^2}{36}$

Thus the fraction of the field that can be grazed

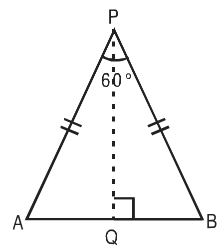
$= \frac{26}{36}\left(\frac{\text{Area that can be grazed}}{\text{Area of the field}}\right)$

The fraction that cannot be grazed = $\frac{10}{36}$

= 28% (approx.)

40. a It is very clear, that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle.
Hence the required ratio = 1 : 12

41. b



Given $\angle APB = 60°$ and AB = b.

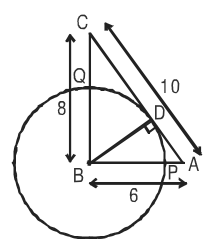
$\therefore PQ = \frac{b}{2} \times \sqrt{3}$

Next, $\frac{b}{2}$, h and PQ form a right angle triangle.

$\therefore \frac{b^2}{4} + h^2 = \frac{3b^2}{4}$

$\therefore 2h^2 = b^2$

42. d



Triangle ABC is a right angled triangle.

Thus $\frac{1}{2}$ × BC × AB = $\frac{1}{2}$ × BD × AC

Or, 6 × 8 = BD × 10. Thus BD = 4.8.
Therefore, BP = BQ = 4.8.
So, AP = AB – BP = 6 – 4.8 = 1.2 and CQ = BC – BQ = 8 – 4.8 = 3.2.
Thus, AP : CQ = 1.2 : 3.2 = 3 : 8

43. b Using the Basic Proportionality Theorem, $\frac{AB}{PQ} = \frac{BD}{QD}$

and $\frac{PQ}{CD} = \frac{BQ}{BD}$.

Multiplying the two we get, $\frac{AB}{CD} = \frac{BQ}{QD} = 3 : 1$.

Thus CD : PQ = BD : BQ = 4 : 3 = 1 : 0.75

44. a If y = 10°,

$\angle BOC = 10°$ (opposite equal sides)

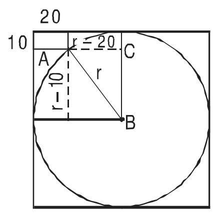
$\angle OBA = 20°$ (external angle of ΔBOC)

$\angle OAB = 20$ (opposite equal sides)

$\angle AOD = 30°$ (external angle of ΔAOC)

Thus k = 3

45. c



Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have $(r - 10)^2 + (r - 20)^2 = r^2$
i.e. $r^2 - 60r + 500 = 0$. Thus r = 10 or 50.
It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

For questions 46 to 48: $A_1A_2 = 2r$, $B_1B_2 = 2r + r\sqrt{3}$, $C_1C_2 = 2r + 2r\sqrt{3}$

Hence, a = 3 × 2r

b = 3 × $(2r + r\sqrt{3})$

c = 3 × $\left(2r + 2r\sqrt{3}\right)$

46. a Difference between (1) and (2) is $3\sqrt{3}r$ and that between (2) and (3) is $3\sqrt{3}r$. Hence, (1) is the correct choice.

47. c Time taken by A = $\frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \left(\frac{2r \times 9}{60}\right) = \frac{3}{10}r$

Therefore, B and C will also travel for time $\frac{3}{10}r$.

Now speed of B = $\left(10\sqrt{3} + 20\right)$

Therefore, the distance covered

$= \left(10\sqrt{3} + 20\right) \times \frac{3}{10}r = \left(\sqrt{3} + 2\right) \times 10 \times \frac{3}{10}r$

$= \left(2r + \sqrt{3}r\right) \times 3 = B_1B_2 + B_2B_3 + B_3B_1$

$\therefore$ B will be at B_1.
Now time taken by for each distance are

$$\frac{C_1C_2}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_2C_3}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_3C_1}{120}$$

$$\frac{3}{40} \times \frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{3}{40} \times \frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{\left(2+2\sqrt{3}\right)r}{120}$$

i.e. $\frac{3}{40} \times 2r, \frac{3}{40} \times 2r, \frac{\left(1+\sqrt{3}\right)}{60}r$

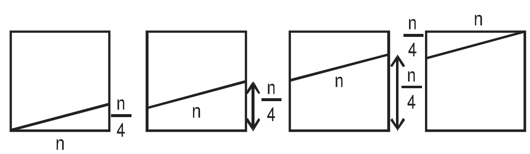
i.e. $\frac{3}{20}r, \frac{3}{20}r, \frac{\left(1+\sqrt{3}\right)}{60}r$

We can observe that time taken for C_1C_2 and C_2C_3 combined is $\frac{3}{20}r + \frac{3}{20}r = \frac{3}{10}r$, which is same as time taken by A. Therefore, C will be at C_3.

48. b In similar triangles, ratio of Area = Ratio of squares of corresponding sides.
Hence, A and C reach A_3 and C_3 respectively.

49. a The whole height h will be divided into n equal parts. Therefore, spacing between two consecutive turns $= \frac{h}{n}$.

50. b The four faces through which string is passing can be shown as



Therefore, length of string in each face

$= \sqrt{n^2 + \left(\frac{n}{4}\right)^2}$

$= \sqrt{n^2 + \frac{n^2}{16}} = \frac{\sqrt{17}n}{4}$

Therefore, length of string through four faces

$= \frac{\sqrt{17}n}{4} \times 4 = \sqrt{17}n$

51. c As h/n = number of turns = 1 (as given). Hence h = n.

52. c PQ || AC

$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$

QD || PC

$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

As $\frac{PD}{DB} = \frac{4}{3}$

$\therefore PD = \frac{4}{7}PB$

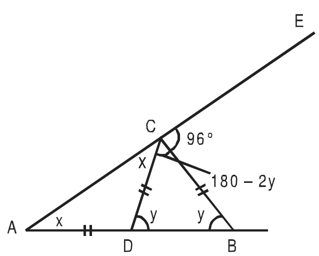
$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB}$

$= \frac{7}{4} \times \frac{AP}{PB}$

$= \frac{7}{4} \times \frac{4}{3}$

= 7 : 3

53. c



Using exterior angle theorem

$\angle A + \angle B = 96$

i.e. x + y = 96 … (i)

Also x + (180 – 2y) + 96 = 180°

$\therefore$ x – 2y + 96 = 0

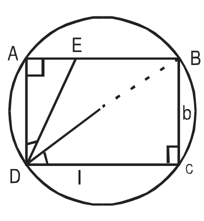
$\therefore$ x – 2y = –96 … (ii)

Solving (i) and (ii),

y = 64° and x = 32°

$\therefore \angle DBC = y = 64^\circ$

54. a



BD = 2r

$$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi r^2}{lb} = \frac{\pi}{\sqrt{3}}$$

$$\frac{r^2}{lb} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{d^2}{4}}{lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{d^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2 + b^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2 + b^2}{lb} = \frac{4}{\sqrt{3}}$$

$$\therefore \frac{l}{b} + \frac{b}{l} = \frac{4}{\sqrt{3}} \quad \text{… (i)}$$

Now Δ AEB ~ Δ CBD

$$\therefore \frac{AE}{CB} = \frac{AD}{DC}$$

$$\therefore \frac{AE}{AD} = \frac{BC}{DC}$$

$$\therefore \frac{AE}{AD} = \frac{b}{l}$$

$\therefore$ We have to find $\frac{AE}{AD}$, i.e. $\frac{b}{l}$.

Let $\frac{b}{l} = x$

Therefore, from (i), we get

$$\frac{1}{x} + x = \frac{4}{\sqrt{3}}$$

$$\frac{1 + x^2}{x} = \frac{4}{\sqrt{3}}$$

$$\sqrt{3} + \sqrt{3}x^2 = 4x$$

$$\therefore \sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\therefore x = \frac{-(-4) \pm \sqrt{16 - 4\left(\sqrt{3}\right)\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2\sqrt{3}}$$

$$= \frac{4 \pm 2}{2\sqrt{3}}$$

$= \frac{6}{2\sqrt{3}}$

OR $\frac{2}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{1}$ OR $\frac{1}{\sqrt{3}}$

From options, the answer is $\frac{1}{\sqrt{3}}$, i.e. $1:\sqrt{3}$.

55. c It's standard property among circle, square and triangle, for a given parameter, area of circle is the highest and area of the triangle is least whereas area of the square is in-between, i.e. c > s > t.

56. c $$\frac{P + \frac{P}{\sqrt{2}} + \cdots\infty}{A + \frac{A}{2} + \cdots\infty} = \frac{\frac{P}{1 - \frac{1}{\sqrt{2}}}}{2A} = \frac{P\sqrt{2}}{\left(\sqrt{2} - 1\right)} \times \frac{1}{2A}$$

$$= \frac{\sqrt{2}P\,(\sqrt{2}+1)}{2A} \qquad = \frac{\sqrt{2} \times 4a\left(\sqrt{2}+1\right)}{2 \times a^2}$$

$$= \frac{\sqrt{2} \times 2\left(\sqrt{2}+1\right)}{a} = \frac{2\left(2+\sqrt{2}\right)}{a}$$

57. a $\angle BAC = \angle ACT + \angle ATC = 50 + 30 = 80^\circ$

And $\angle ACT = \angle ABC$ (Angle in alternate segment)

So $\angle ABC = 50^\circ$

$\angle BCA = 180 - \left(\angle ABC + \angle BAC\right)$

$= 180 - \left(50 + 80\right) = 50^\circ$

Since $\angle BOA = 2\angle BCA = 2 \times 50 = 100^\circ$

Alternative Method:

Join OC

$\angle OCT = 90^\circ$ (TC is tangent to OC)

$\angle OCA = 90^\circ - 50^\circ = 40^\circ$

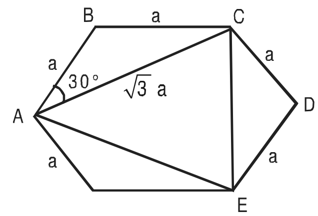
$\angle OAC = 40^\circ$ (OA = OC being the radius)

$\angle BAC = 50^\circ + 30^\circ = 80^\circ$

$\angle OAB = 80^\circ - 40^\circ = 40^\circ = \angle OBA$ (OA = OB being the radius)

$\angle BOA = 180^\circ - (\angle OBA + \angle OAB) = 100^\circ$

58. b



$\because \Delta$ ACE is equilateral triangle with side $\sqrt{3}$ a.

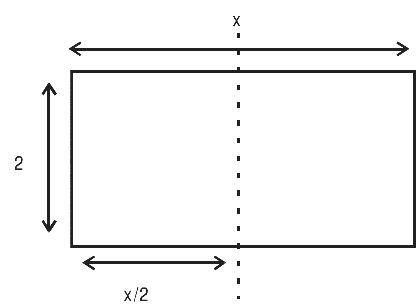
Area of hexagon = $\frac{\sqrt{3}}{4}\ a^2 \times 6$

Area as $\Delta ACE = \frac{\sqrt{3}}{4}\ (\sqrt{3}a)^2$

Therefore, ratio = $\frac{1}{2}$

59. d The required answer is 34 × 0.65 × 0.65 = 14.365 Because we get two similar triangles and area is proportional to square of its side.

60. b



In original rectangle ratio = $\frac{x}{2}$

In Smaller rectangle ratio = $\frac{2}{\left(\frac{x}{2}\right)}$

Given $\frac{x}{2} = \frac{2}{\frac{x}{2}} \Rightarrow x = 2\sqrt{2}$

Area of smaller rectangle = $\frac{x}{2} \times 2 = x = 2\sqrt{2}$ sq. units

61. b $\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

$OP = 28$
$OQ = 21$
$PQ = OP - OQ = 7$

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

62. b $PR + QS = PQ = 7$

$= \frac{PR}{QS} = \frac{4}{3}$

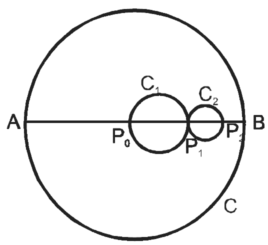
$\Rightarrow QS = 3$

63. c $SO = \sqrt{OQ^2 - QS^2}$

$= \sqrt{21^2 - 3^2}$

$= \sqrt{24 \times 18} = 12\sqrt{3}$

64. d



Circle	Radius
C	r
C_1	$\frac{r}{4}$
C_2	$\frac{r}{8}$
C_3	$\frac{r}{16}$
⋮	⋮

$\Rightarrow$ either $\frac{\text{Area of unshaded portion of C}}{\text{Area of C}}$

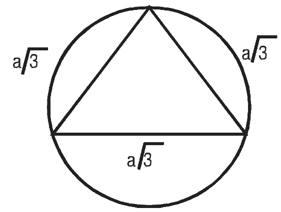
$= 1 - \frac{\text{Area of shaded portion}}{\text{Area of C}}$

$= 1 - \frac{\pi\left(\left(\frac{r}{4}\right)^2 + \left(\frac{r}{8}\right)^2 + \ldots\right)}{\pi r^2}$

$= 1 - \left(\frac{1}{4^2} + \frac{1}{8^2} + \ldots\right) = 1 - \frac{\frac{1}{16}}{1 - \frac{1}{4}}$

$= \frac{11}{12}$

65. a DF, AG and CE are body diagonals of cube.
Let the side of cube = a
Therefore body diagonal is $a\sqrt{3}$

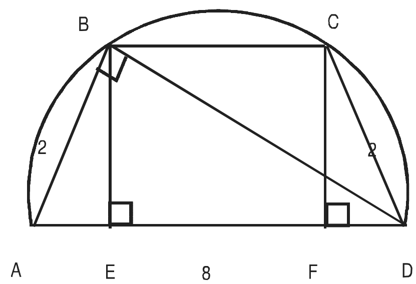


Circum radius for equilateral triangle

$= \frac{\text{side}}{\sqrt{3}}$

Therefore $\frac{a\sqrt{3}}{\sqrt{3}} = a$

66. b



$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$

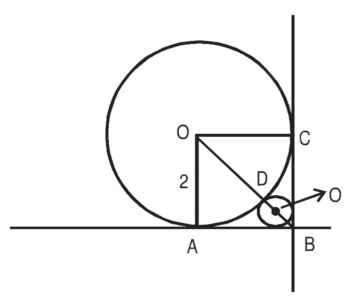
$2\sqrt{8^2 - 2^2} = 8 \times BE$

$BE = \frac{\sqrt{60}}{4} = \frac{\sqrt{15}}{2}$

$AE = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$

$BC = EF = 8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$

67. d



Let the radius of smaller circle = r

$\therefore O'B = r\sqrt{2}$

$\therefore OB = O'B + O'D + OD$

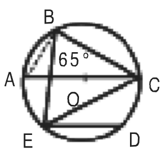
$= r\sqrt{2} + r + 2$

Also OB = $2\sqrt{2}$

$\Rightarrow r\sqrt{2} + r + 2 = 2\sqrt{2}$

$\Rightarrow r = 6 - 4\sqrt{2}$

68. d



In ΔABC,

$\angle B = 90°$ (Angles in semicircle)

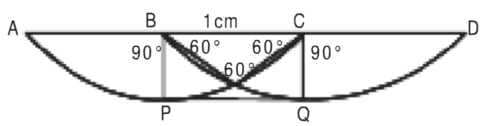
Therefore $\angle ABE = 90 - 65 = 25°$

Also $\angle ABE = \angle ACE$ (angle subtended by same arc AE)

Also $\angle ACE = \angle CED$ [AC ∥ ED]

Therefore $\angle CED = 25°$

69. b



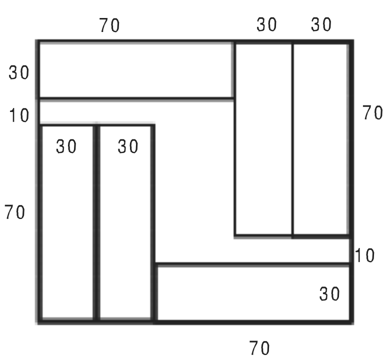
Drawn figure since it have not to be within distance of 1 cm so it will go along APQD.

AP = $\frac{90}{360} \times 2\pi \times 1 = \frac{\pi}{2}$

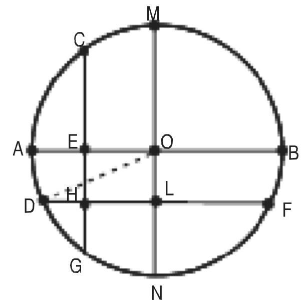
Also AP = QD = $\frac{\pi}{2}$

So the minimum distance = AP + PQ + QD = $\frac{\pi}{2} + 1 + \frac{\pi}{2} = 1 + \pi$

70. c



71. b



$HL = OE = \frac{1}{2}$

$DL = DH + HL$

$DL = DH + \frac{1}{2}$

OB = AO = radius = 1.5

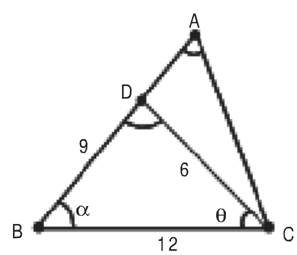
$DO^2 = OL^2 + DL^2$

$\left(\frac{3}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(DH + \frac{1}{2}\right)^2$

$\Rightarrow \left(DH + \frac{1}{2}\right)^2 = 2 \Rightarrow DH = \sqrt{2} - \frac{1}{2}$

Hence option (b)

72. a



Here $\angle ACB = \theta + 180 - (2\theta + \alpha) = 180 - (\theta + \alpha)$

So here we can say that triangle BCD and triangle ABC will be similar.

Hence from the property of similarity

$\frac{AB}{12} = \frac{12}{9}$ Hence AB = 16

$\frac{AC}{6} = \frac{12}{9}$ Hence AC = 8

Hence AD = 7

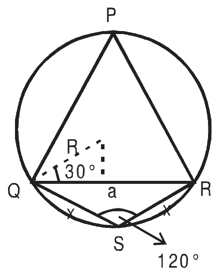
AC = 8

$S_{ADC} = 8 + 7 + 6 = 21$

$S_{BDC} = 27$

Hence $r = \frac{21}{27} = \frac{7}{9}$

73. a



Here $\cos 30° = \frac{a}{2r}$

$a = r\sqrt{3}$

Here the side of equilateral triangle is $r\sqrt{3}$

From the diagram $\cos 120° = \frac{x^2 + x^2 - a^2}{2x^2}$

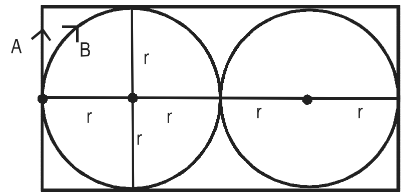
$a^2 = 3x^2$
$x = r$

Hence the circumference will be $2r\left(1+\sqrt{3}\right)$

Hence answer is (a).

74. b Let the rectangle has m and n tiles along its length and breadth respectively.
The number of white tiles
$W = 2m + 2(n - 2) = 2(m + n - 2)$
And the number of Red tiles $= R = mn - 2(m + n - 2)$
Given $W = R \Rightarrow 4(m + n - 2) = mn$
$\Rightarrow mn - 4m - 4n = -8$
$\Rightarrow (m - 4)(n - 4) = 8$
$\Rightarrow m - 4 = 8$ or $4 \Rightarrow m = 12$ or 8
$\therefore$ 12 suits the options.

75. d



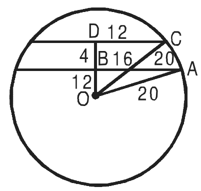
A covers 2r + 2r + 4r + 4r = 12 r
B covers $2\pi r + 2\pi r = 4\pi r$ distance

$\frac{4\pi r}{S_B} = \frac{12r}{S_A} \Rightarrow S_B = \frac{\pi}{3} S_A$

$\frac{S_B - S_A}{S_A} \times 100 = \frac{\pi - 3}{3} \times 100 = 4.72\%$

Hence Option (d)

76. d



$OB^2 = OA^2 - AB^2 = 20^2 - 16^2 = 144$
$OB = 12$
$OD^2 = 20^2 - 12^2 = 400 - 144 = 256$
$OD = 16$
$BD = 4$
Only one option contains 4 hence other will be 28.
Hence option (d)